Monopoly and cournot

1. Suppose a market where there is a single supplier with production costs given by $C(q) = cq$ and its inverse demand function $P(q) = 4 - q$. Remembering that $Q = q$.

   Determine the market structure that arises from maximizing benefits by finding the quantities to produce and the optimal price.

2. Suppose two firms have asymmetric costs ($C_1 = 2$, $C_2 = 3$) and compete (with quantities) in a market where the inverse demand is: $P = 4 - Q$, $Q = q_1 + q_2$.

Solution

1. We set up the profit maximization,

$$\pi(q) = p(q) \cdot q - c(q)$$

$$\pi(q) = (4 - q)q - cq$$

$$\pi(q) = 4q - q^2 - cq$$

First-order condition (FOC):

$$\frac{\partial \pi}{\partial q} = 4 - 2q - c = 0$$

Solving for q,

$$q^* = \frac{4 - c}{2}$$

To find the price, we substitute into the inverse demand function the quantities found,

$$P(q) = 4 - \left(\frac{4 - c}{2}\right)$$

$$P^* = 2 + \frac{1}{2}c$$

2. We set up the profit for both firms and differentiate concerning their quantities:

$$\Pi_1 = (4 - q_1 - q_2 - 2)q_1$$

$$\Pi_2 = (4 - q_1 - q_2 - 3)q_2$$

$$\frac{\partial \Pi_1}{\partial q_1} = 2 - 2q_1 - q_2 = 0$$

$$\frac{\partial \Pi_2}{\partial q_2} = 1 - 2q_2 - q_1 = 0$$

We derive the reaction functions from the derivatives:

$$q_1^{FR} = \frac{2 - q_2}{2}$$

$$q_2^{FR} = \frac{1 - q_1}{2}$$

We equate the reaction functions:

$$q_1 = 1 - \frac{1}{2}\left(\frac{1}{2} - q_1/2\right)$$
$$q_1 = 1 - \left(\frac{1}{4} - q_1/4\right)$$
$$\frac{3}{4}q_1 = 3/4$$

$$q_1^* = 1$$

$$q_2^* = 0$$

$$Q = 1 \Rightarrow p = 3$$

$$\Pi_1 = (p-c)q_1 = (3-2)\cdot 1 = 1$$

$$\Pi_2 = 0$$